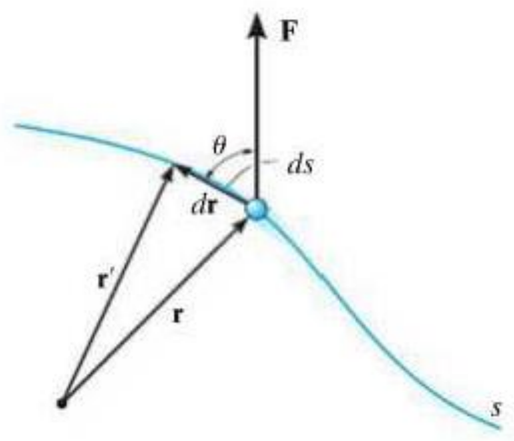
WEEK 5

GEC 241- APPLIED MECHANICS II- DYNAMICS
KINETICS OF PARTICLES

The work of a Force

Specifically, a force **F** will do work on a particle only when the particle undergoes a displacement in the direction of the force. In the diagram below, the force **F** causes the particle to move along the path s from position **r** to a new position **r’,** the displacement is then dr = r’ – **r.** The magnitude of dr is ds, the length of the differential segment along the path. If the angle between the tails of dr and **F** is θ, then the work done by **F** is a scalar quantity defined by



$$dU = F\ ds\ cos\theta \qquad (1)$$

By definition of the dot product this equation can be written as

$$dU = F.dr \qquad (2)$$

This result may be interpreted in one of two ways: either as the product of F and the component of displacement ds cos θ in the direction of the force, or as the product of ds and the component of force, Fcos θ, in the direction of the displacement. Note that if $0^0 \leq \theta < 90^0$, then the force component and displacement have the same sense so that the work is positive, whereas if $90^0 < \theta \leq 180^0$, these vectors will have opposite sense and therefore the work is negative. Also dU = 0, or if the force is perpendicular to displacement, since cos 90^0= 0, or if the force is applied at a fixed point in which the displacement is zero.

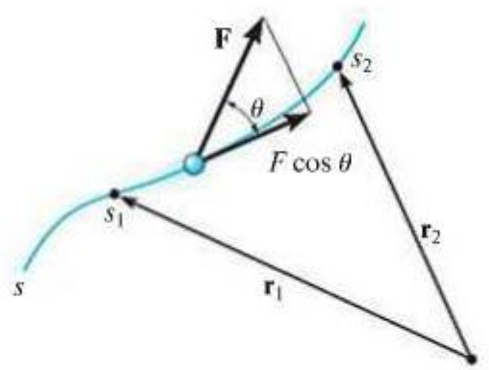
The unit of work in SI units is the joule (J), which is the amount of work done by a one-newton force when it moves through a distance of one meter in the direction of the force (1 J = 1 N.m).

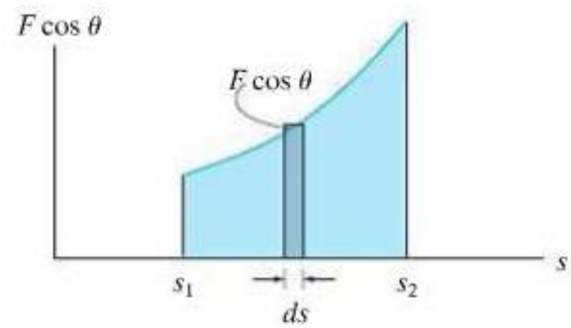
Work of a variable force

If the particle acted upon by the force F undergoes a finite displacement along its path from r_1 to r_2 or s_1 to s_2, the work of force F is determine by integration. Provided F and θ can be expressed as a function of position, then

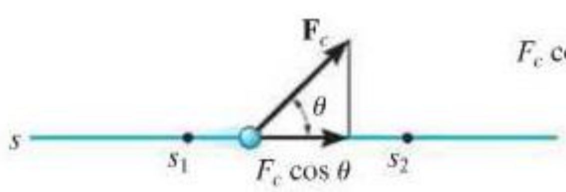
$$U_{1-2} = \int_{r_1}^{r_2} F.dr = \int_{r_1}^{r_2} Fcos\theta ds \qquad (3)$$

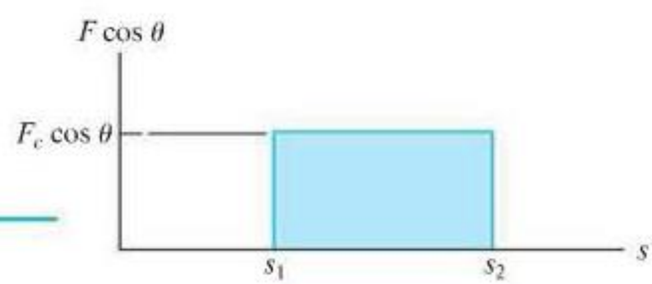
Sometimes, this relation may be obtained by using experimental data to plot a graph of Fcos θ vs s. then the area under this graph bounded by s_1 and s_2 represents the total work.





Work of a constant force moving along a straight line





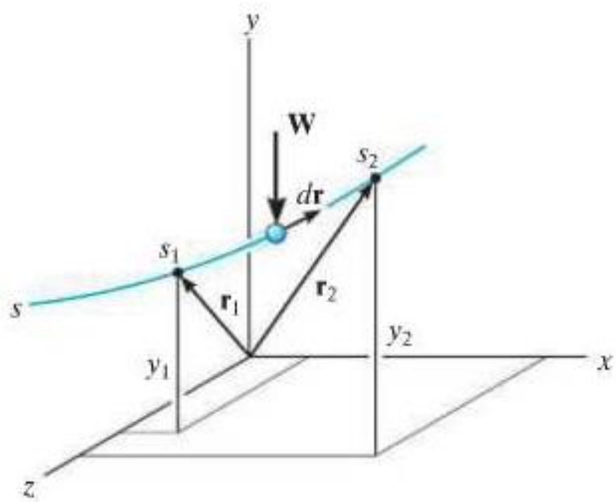
If the force F_c has a constant magnitude and acts at a constant angle θ from its straight line path, then the component of F_c in the direction of displacement is always $F_c$cos θ. The work done by F_c when the particle is displaced from s_1 to s_2 is determined in which case

$$U_{1-2} = F_c cos\theta \int_{s_1}^{s_2} ds$$

Or

$$U_{1-2} = F_c cos\theta(s_2 - s_1) \qquad (3)$$

Work of a weight



Consider a particle of weight W, which moves up along the path s from position s_1 to position s_2. At an intermediate point, the displacement $dr = dx\boldsymbol{i} + dy\boldsymbol{j} + dz\boldsymbol{k}$. Since W = -W**j**

$$U_{1-2} = \int F.dr = \int_{r_1}^{r_2} (-Wj) \cdot (dxi + dyj + dzk)$$

$$= \int_{y_1}^{y_2} -Wdy = -W(y_2 - y_1)$$

Or

$$U_{1-2} = -W\Delta y \qquad (4)$$

Thus the work is independent of the path and is equal to the magnitude of the particle’s weight times its vertical displacement.

Work of a spring force

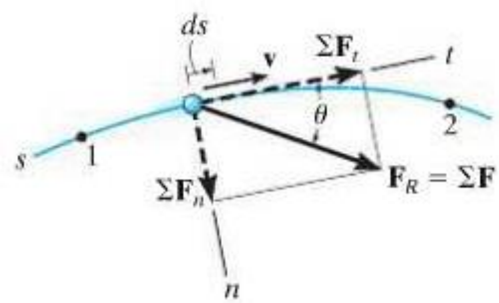
If an elastic spring is elongated a distance ds, then the work done by the force that acts on the attached particle is $dU = -F_s ds = -ks\ ds$. The work is negative since F_s acts in the opposite sense to ds. If the particle displaces from s_1 to s_2, the work of F_s is then

$$U_{1-2} = \int_{s_1}^{s_2} F_s ds = \int_{s_1}^{s_2} -ks\ ds$$

$$U_{1-2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right) \qquad (5)$$

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle- if both are in the same sense, positive work results; if they are opposite to one another, the work is negative.

Principle of work and energy



Consider a particle of mass *m* acted upon by a force **F** and moving along a path which is either rectilinear or curved. Expressing Newton's second law in terms of the tangential components of the force and of the acceleration, we write

$$F_t = ma_t \quad or \quad F_t = m\frac{dv}{dt}$$

where v is the speed of the particle. Recalling that $v = ds/dt$, we obtain

$$F_t = m\frac{dv\ ds}{ds\ dt} = mv\frac{dv}{ds}$$

$$F_t ds = mvdv \qquad (6)$$

Integrating from A_1 where $s=s_1$ and $v=v_1$, to A_2, where $s = s_2$ and $v = v_2$, we write

$$\int_{s_1}^{s_2} F_t ds = m\int_{v_1}^{v_2} vdv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad (7)$$

The left-hand member of the above equation represents the work of the force **F** exerted on the particle during the displacement from A_1 to A_2 as indicated, the work is a scalar quantity. The expression $\frac{1}{2}mv^2$is also a scalar quantity; it is defined as the kinetic energy of the particle and is denoted by *T*. We write

$$T = \frac{1}{2}mv^2$$

Substituting into (7), we have

$$U_{1-2} = T_2 - T_1 \qquad (8a)$$

which expresses that, when a particle moves from A_1 to A_2 under the action of a force **F**, *the work of the force* **F** *is equal to the change in kinetic energy of the particle*. This is known as the *principle of work and energy*. Rearranging the terms, we write

$$T_1 + U_{1-2} = T_2 \qquad (8b)$$

Thus, *the kinetic energy of the particle at A_2 can be obtained by adding to its kinetic energy at A_1 the work done during the displacement from A_1 to A_2 by the force* **F** *exerted on the particle*. Like Newton's second law from which it is derived, the principle of work and energy applies only with respect to a newtonian frame of reference. The speed *v* used to determine the kinetic energy *T* should therefore be measured with respect to a newtonian frame of reference.

Since both work and kinetic energy are scalar quantities, their sum can be computed as an ordinary algebraic sum, the work U_{1-2} being considered as positive or negative according to the direction of F. When several forces act on the particle, the expression U_{1-2} represents the total work of the forces acting on the particle; it is obtained by adding algebraically the work of the various forces.

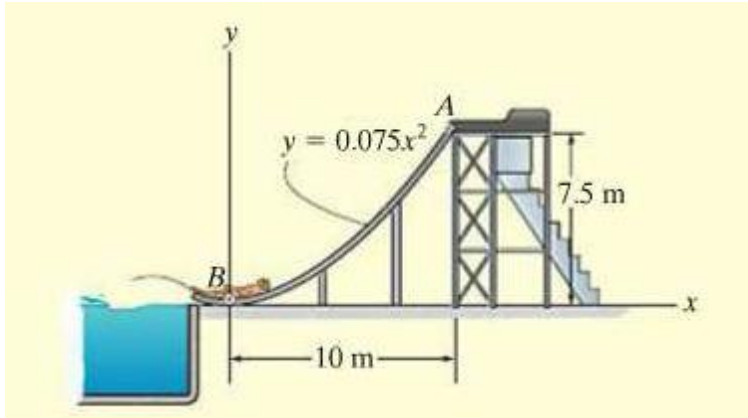
As noted above, the kinetic energy of a particle is a scalar quantity. It further appears from the definition $T = \frac{1}{2}mv^2$ that regardless of the direction of motion of the particle the kinetic energy is always positive. Considering the particular case when v_1 = 0 and v_2 = *v*, and substituting T_1 = 0 and T_2 = *T into eq. (8b)*, we observe that the work done by the forces acting on the particle is equal to *T*. Thus, the kinetic energy of a particle moving with a speed *v* represents the work which must be done to bring the particle from rest to the speed *v*. Substituting

T_1 = *T* and T_2 = 0 into eq.(8b), we also note that when a particle moving with a speed *v* is brought to rest, the work done by the forces acting on the particle is 2*T*. Assuming that no energy is dissipated into heat, we conclude that the work done by the forces exerted *by the particle* on the bodies which cause it to come to rest is equal to *T*. Thus, the kinetic energy of a particle also represents *the capacity to do work associated with the speed of the particle*.

The kinetic energy is measured in the same units as work, i.e., in joules if SI units are used and in ft/lb if U.S. customary units are used.

Example1

The 40kg boy in the figure below slides down the smooth water slide. If he starts from rest at A, determine his speed when he reaches B and the normal reaction the slide exerts on the boy at this position.



Solution

Using the principle of work and energy,

$$T_A + U_{A-B} = T_B$$

$$0 + (40(9.81)N)7.5m) = \frac{1}{2}(40kg)v_B^2$$

$$v_B = 12.13\, m/s$$

When the boy is at B, the normal reaction N_B can be obtained by applying the equation of motion along the n-axis. Here the radius of curvature of the path is

$$\rho_B = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (0.15x)^2\right]^{3/2}}{|0.15|}\right|_{x=0} = 6.667 \text{ m}$$

Thus,

$$+\uparrow \Sigma F_n = ma_n; \qquad N_B - 40(9.81)\text{ N} = 40\text{ kg}\left(\frac{(12.13\text{ m/s})^2}{6.667\text{ m}}\right)$$

$$N_B = 1275.3\text{ N} = 1.28\text{ kN}$$

Power and Efficiency

Power is defined as the time rate at which work is done. In the selection of a motor or engine , power is much more important criterion than is the actual amount of work to be performed. The higher the power of an engine, the faster it completes required task.

The power generated by a machine or engine that performs an amount of work dU within the time interval dt is

$$P = \frac{dU}{dt} \qquad (9)$$

If the work dU is expressed as dU = F.dr, then

$$P = \frac{dU}{dt} = \frac{F.dr}{dt} = F.\frac{dr}{dt}$$

Or

$$P = F.v \qquad (10)$$

The basic unit of power is Watt. Sometime horsepower can be used.
Note that 1 hp = 746 W.
Efficiency of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\varepsilon = \frac{power\ output}{power\ input} \qquad (11)$$

If energy supplied to the machine occurs during the same time interval at which it is drawn, then the efficiency may also be expressed in terms of the ratio.

$$\varepsilon = \frac{energy\ output}{energy\ input}$$

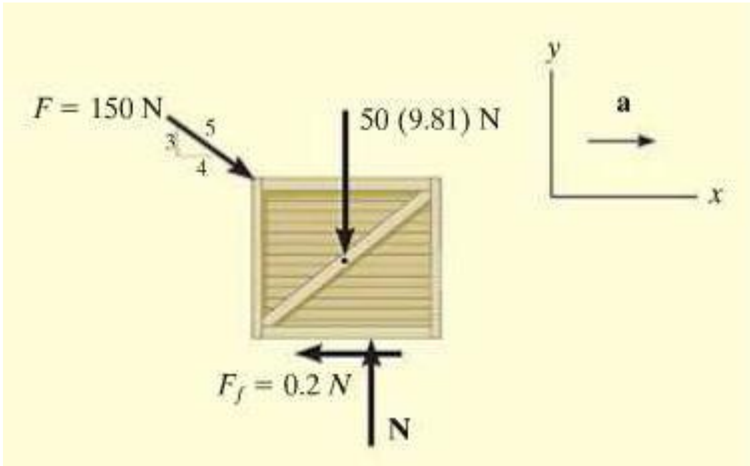
Since machines consist of series of moving parts, frictional forces will always be developed within the machine and as a result, extra energy or power is needed to overcome these forces. Consequently, power output will be less than power input and o the efficiency f a machine is always less than 1.

Example2

The man in the figure pushes on the 50kg crate with a force of F=150 N. Determine the power supplied by the man when t = 4s. The coefficient of kinetic friction between the floor and the crate is $\mu_k = 0.2$. Initially the crate is at rest.

Solution

To determine the power developed by the man, the velocity of the 150N force must be obtained first. Applying the equation of motion,



$$+\uparrow \Sigma F_y = ma_y; \qquad N - \left(\tfrac{3}{5}\right)150\text{ N} - 50(9.81)\text{ N} = 0$$

$$N = 580.5\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad \left(\tfrac{4}{5}\right)150\text{ N} - 0.2(580.5\text{ N}) = (50\text{ kg})a$$

$$a = 0.078\text{ m/s}^2$$

The velocity of the crate when $t = 4$ s is therefore

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v = 0 + (0.078\text{ m/s}^2)(4\text{ s}) = 0.312\text{ m/s}$$

The power supplied to the crate by the man when $t = 4$ s is therefore

$$P = \mathbf{F}\cdot\mathbf{v} = F_x v = \left(\tfrac{4}{5}\right)(150\text{ N})(0.312\text{ m/s})$$

$$= 37.4\text{ W} \qquad \textit{Ans.}$$

Principle of impulse and momentum

Consider a particle of mass m acted upon by a force F. Using he Newton's second law, the force can be expressed as

$$F = \frac{d}{dt}(mv)$$

where mv is the linear momentum of the particle. Multiplying both sides by dt and integrating from a time t_1 to a time t_2, we have

$$Fdt = d(mv)$$

$$\int_{t_1}^{t_2} Fdt = mv_2 - mv_1$$

or, transposing the last term,

$$mv_1 + \int_{t_1}^{t_2} F dt = mv_2 \qquad (12)$$

The integral is a vector known as the linear impulse, or simply the impulse of the force F during the interval of time considered.

$$mv_1 + Imp_{1-2} = mv_2 \qquad (13)$$

When several forces act on a particle, the impulse of each of the forces must be considered. We have

$$mv_1 + \sum Imp_{1-2} = mv_2 \qquad (14)$$

If no external force is exerted on the particles or more generally, if the sum of the external forces is zero, the second term in the above equation vanishes and reduces to

$$\sum mv_1 = \sum mv_2 \qquad (15)$$

which expresses that the total momentum of the particles is considered.

Example 3

The bumper cars A and B in Fig. 15–9a each have a mass of 150 kg and are coasting with the velocities shown before they freely collide head on. If no energy is lost during the collision, determine their velocities after collision.

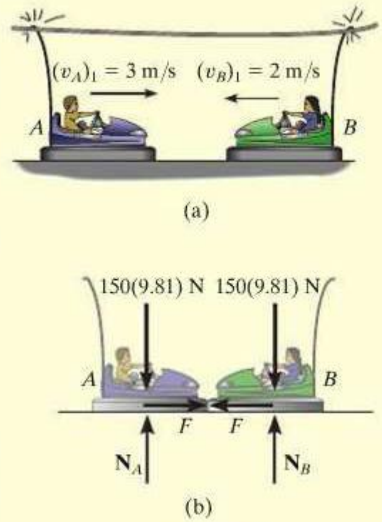


Fig. 15–9

SOLUTION

Free-Body Diagram. The cars will be considered as a single system. The free-body diagram is shown in Fig. 15–9b.

Conservation of Momentum.

$$(\xrightarrow{+}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(150\text{ kg})(3\text{ m/s}) + (150\text{ kg})(-2\text{ m/s}) = (150\text{ kg})(v_A)_2 + (150\text{ kg})(v_B)_2$$

$$(v_A)_2 = 1 - (v_B)_2 \qquad (1)$$

Conservation of Energy. Since no energy is lost, the conservation of energy theorem gives

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2 + 0 = \frac{1}{2}m_A(v_A)_2^2 + \frac{1}{2}m_B(v_B)_2^2 + 0$$

$$\frac{1}{2}(150\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(150\text{ kg})(2\text{ m/s})^2 + 0 = \frac{1}{2}(150\text{ kg})(v_A)_2^2 + \frac{1}{2}(150\text{ kg})(v_B)_2^2 + 0$$

$$(v_A)_2^2 + (v_B)_2^2 = 13 \qquad (2)$$

Substituting Eq. (1) into (2) and simplifying, we get

$$(v_B)_2^2 - (v_B)_2 - 6 = 0$$

Solving for the two roots,

$$(v_B)_2 = 3\text{ m/s} \qquad \text{and} \qquad (v_B)_2 = -2\text{ m/s}$$

Since $(v_B)_2 = -2$ m/s refers to the velocity of B just *before* collision, then the velocity of B just after the collision must be

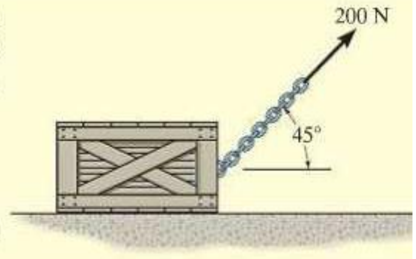
$$(v_B)_2 = 3\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

Substituting this result into Eq. (1), we obtain

$$(v_A)_2 = 1 - 3\text{ m/s} = -2\text{ m/s} = 2\text{ m/s} \leftarrow \qquad \textit{Ans.}$$

Example 4

The 100-kg crate shown in Fig. 15–4*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied for 10 s, determine the final velocity and the normal force which the surface exerts on the crate during this time interval.

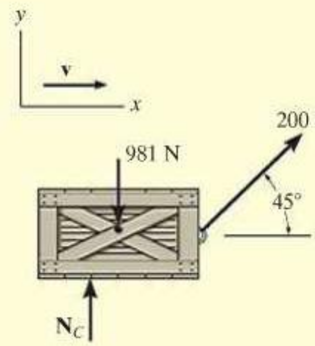


(a)

SOLUTION

This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

Free-Body Diagram. See Fig. 15–4*b*. Since all the forces acting are *constant*, the impulses are simply the product of the force magnitude and 10 s $[\mathbf{I} = \mathbf{F}_c(t_2 - t_1)]$. Note the alternative procedure of drawing the crate's impulse and momentum diagrams, Fig. 15–4*c*.



(b)

Principle of Impulse and Momentum. Applying Eqs. 15–4 yields

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$0 + 200 \text{ N} \cos 45°(10 \text{ s}) = (100 \text{ kg})v_2$$

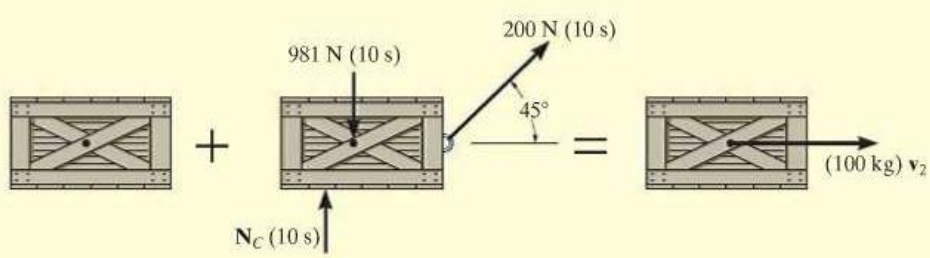
$$v_2 = 14.1 \text{ m/s} \qquad \textit{Ans.}$$

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$0 + N_C(10 \text{ s}) - 981 \text{ N}(10 \text{ s}) + 200 \text{ N} \sin 45°(10 \text{ s}) = 0$$

$$N_C = 840 \text{ N} \qquad \textit{Ans.}$$

NOTE: Since no motion occurs in the *y* direction, direct application of the equilibrium equation $\Sigma F_y = 0$ gives the same result for N_C. Try to solve the problem by first applying $\Sigma F_x = ma_x$, then $v = v_0 + a_c t$.



(c)

Impulsive motion

A force acting on a particle during a very short time interval that is large enough to produce a definite change in momentum is called an *impulsive force* and the resulting motion is called an *impulsive motion*. For example, when a baseball is struck, the contact between bat and ball

takes place during a very short time interval Δt. But the average value of the force **F** exerted by the bat on the ball is very large, and the resulting impulse **F** Δt is large enough to change the sense of motion of the ball.
When impulsive forces act on a particle, this equation holds

$$mv_1 + \sum F\Delta t = mv_2 \qquad (16)$$

Any force which is not an impulsive force may be neglected, since the corresponding impulse **F** Δt is very small. *Non-impulsive forces* include the weight of the body, the force exerted by a spring, or any other force which is *known* to be small compared with an impulsive force. Unknown reactions may or may not be impulsive; their impulses should therefore be included as long as they have not been proved negligible. The impulse of the weight of the baseball considered above, for example, may be neglected. If the motion of the bat is analyzed, the impulse of the weight of the bat can also be neglected. The impulses of the reactions of the player's hands on the bat, however, should be included; these impulses will not be negligible if the ball is incorrectly hit.
We note that the method of impulse and momentum is particularly effective in the analysis of the impulsive motion of a particle, since it involves only the initial and final velocities of the particle and the impulses of the forces exerted on the particle. The direct application of Newton's second law, on the other hand, would require the determination of the forces as functions of the time and the integration of the equations of motion over the time interval Δt.
In the case of the impulsive motion of several particles, it reduces to

$$\sum mv_1 + \sum F\Delta t = \sum mv_2 \qquad (17)$$

where the second term involves only impulsive, external forces. If all the external forces acting on the various particles are non-impulsive, the second term in Eq. (17) vanishes and this equation reduces to Eq. (15). We write

$$\sum mv_1 = \sum mv_2 \qquad (18)$$

which expresses that the total momentum of the particles is conserved. This situation occurs, for example, when two particles which are moving freely collide with one another. We should note, however, that while the total momentum of the particles is conserved, their total energy is generally *not* conserved.

Example 5:
An automobile weighing 4000lb is driven down a 5^0 incline at a speed of 60mi/h when the brakes are applied, causing a constant total braking force (applied by the road on the tires) of 1500lb. Determine the time required for the automobile to come to a stop.

Solution

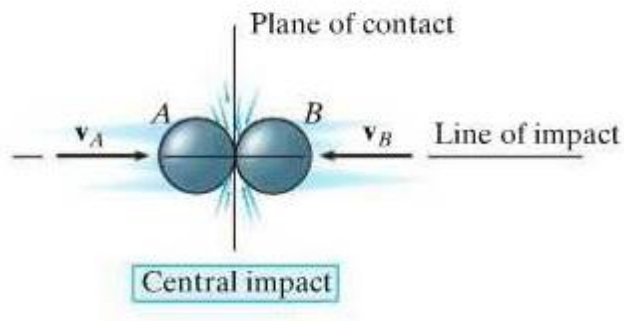
$$mv_1 + \sum Imp_{1-2} = mv_2$$
$$mv_1 + (W sin 5^0)t - Ft = 0$$
$$(4000/32.2)(88) + (4000 sin 5^0)t - 1500t = 0$$
$$t = 9.49\ s$$

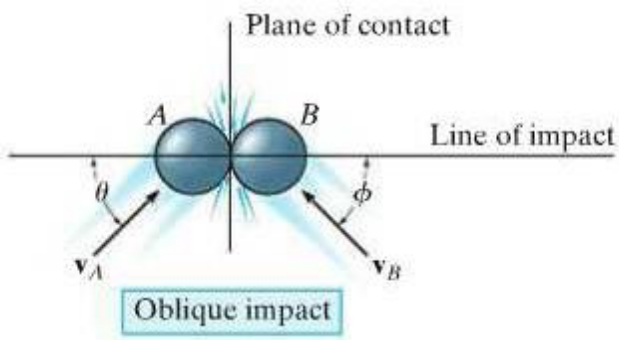
Impact

Impact occurs when two bodies collide with each other during a very short period of time, causing relatively large (impulsive) forces to be exerted between the bodies. The striking of a hammer on a nail, or golf club on a ball etc are examples of impact loading.

In general, there are two types of impact which are central impact and oblique impact.

Central impact occurs when the direction of motion of mass centers of the two colliding particles is along a line passing through the mass centers of the particles. This line is called the line of impact, which is perpendicular to the plane of contact. When the motion of one or both of the particles make an angle with the line of impact, the impact is said to be oblique impact.

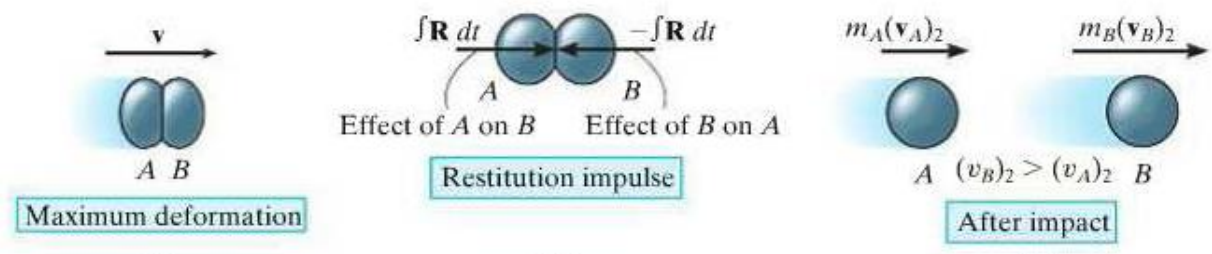




To illustrate the method for analyzing the mechanics of impact, consider the case involving the central impact of the two particles A and B shown in the central impact figure above.

The particles have the initial momenta in which provided $(v_A)_1 > (v_B)_1$ collision will eventually occur. During the collision, the particles must be thought of as deformable or non rigid. The particles will undergo a period of deformation such that they exert an equal but opposite deformation impulse $\int \boldsymbol{P}dt$ on each other. Only at the instant of maximum deformation will both particles move with a common velocity **v**, since their relative motion is zero.

Afterward a period of restitution occurs, in which case the particles will either return to their original shape or remain permanently deformed. The equal but opposite restitution impulse $\int \boldsymbol{R}dt$ pushes the particles apart from one another. In reality, the physical properties of any two bodies are such that the deformation impulse will always be greater than that of restitution, i.e. $\int \boldsymbol{P}dt > \int \boldsymbol{R}dt$**.** Just after separation, the particles will have the final momenta where $(v_B)_2 > (v_A)_2$.



In most problems, the initial velocities of the particles will be known and it will be necessary to determine their final velocities. In this regard, the momentum for the system of particles is conserved since during the internal impulses of deformation and restitution cancel. Hence, we have

$$\left(\overset{+}{\rightarrow}\right) \; m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2 \qquad (19)$$

In order to obtain a second equation necessary to solve for $(v_A)_2$ *and* $(v_B)_2$, the principle of impulse and momentum must be applied to each particle. For instance, during the deformation phase for particle A, we have

$$\left(\overset{+}{\rightarrow}\right) \qquad m_A(v_A)_1 - \int P\,dt = m_A v \qquad (20)$$

For the restitution phase,

$$\left(\overset{+}{\rightarrow}\right) \qquad m_A v - \int R\,dt = m_A(v_A)_2 \qquad (21)$$

The ratio of the restitution impulse to the deformation impulse is called the coefficient of restitution, e. from the above equations, this value for particle A is

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{v - (v_A)_2}{(v_A)_1 - v} \qquad (22)$$

Similarly, we can establish e by considering particle B. this yields

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{(v_B)_2 - v}{v - (v_B)_1} \qquad (23)$$

Eliminating the unknown v from the two equations, the coefficient of restitution can be expressed in terms of the particles' initial and final velocities as

$$(\overset{+}{\rightarrow}) \qquad \boxed{e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}} \qquad (24)$$

Coefficient of restitution

It states that e is equal to the ratio of the relative velocity of particles' separation just after impact to the relative velocity of the particles' approach just before impact.

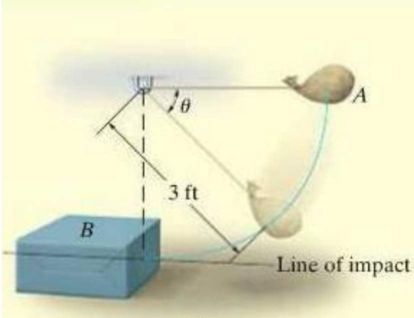
Experiments have proved that e varies appreciably with impact velocity as well as with the size and shape of the colliding bodies. In general e has a value between zero and one.

When e=1, this is called elastic impact. This means that if the collision between two particles is perfectly elastic, the deformation impulse is equal and opposite to the restitution impulse. Although in reality, this can never be achieved.

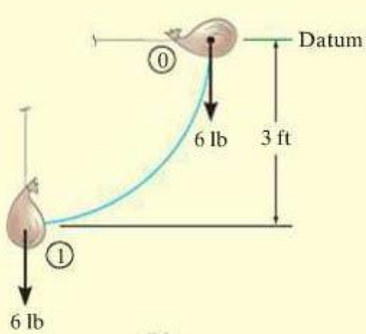
When e=0, the impact is said to be inelastic or plastic. In this case, there is no restitution impulse so that after collision both particles couple or stick together and move with a common velocity.

Note that if the impact is perfectly elastic, no energy is lost in the collision whereas if the collision is plastic, the energy lost during collision is maximum.

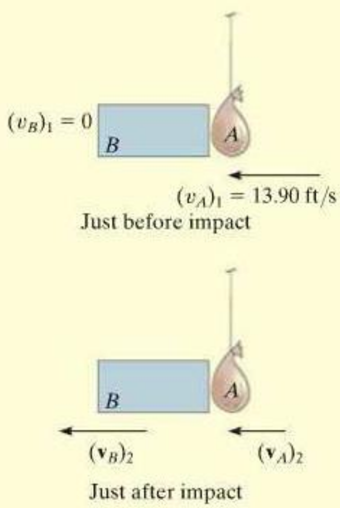
Example 6



(a)



(b)



(c)

The bag A, having a weight of 6 lb, is released from rest at the position $\theta = 0°$, as shown in Fig. 15–16a. After falling to $\theta = 90°$, it strikes an 18-lb box B. If the coefficient of restitution between the bag and box is $e = 0.5$, determine the velocities of the bag and box just after impact. What is the loss of energy during collision?

SOLUTION

This problem involves central impact. Why? Before analyzing the mechanics of the impact, however, it is first necessary to obtain the velocity of the bag *just before* it strikes the box.

Conservation of Energy. With the datum at $\theta = 0°$, Fig. 15–16b, we have

$$T_0 + V_0 = T_1 + V_1$$

$$0 + 0 = \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_1^2 - 6\text{ lb}(3\text{ ft}); (v_A)_1 = 13.90\text{ ft/s}$$

Conservation of Momentum. After impact we will assume A and B travel to the left. Applying the conservation of momentum to the system, Fig. 15–16c, we have

$$(\overset{+}{\leftarrow}) \qquad m_B(v_B)_1 + m_A(v_A)_1 = m_B(v_B)_2 + m_A(v_A)_2$$

$$0 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.90\text{ ft/s}) = \left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_B)_2 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_2$$

$$(v_A)_2 = 13.90 - 3(v_B)_2 \qquad (1)$$

Coefficient of Restitution. Realizing that for separation to occur after collision $(v_B)_2 > (v_A)_2$, Fig. 15–16c, we have

$$(\overset{+}{\leftarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.5 = \frac{(v_B)_2 - (v_A)_2}{13.90\text{ ft/s} - 0}$$

$$(v_A)_2 = (v_B)_2 - 6.950 \qquad (2)$$

Solving Eqs. 1 and 2 simultaneously yields

$$(v_A)_2 = -1.74\text{ ft/s} = 1.74\text{ ft/s} \rightarrow \quad \text{and} \quad (v_B)_2 = 5.21\text{ ft/s} \leftarrow \qquad \textit{Ans.}$$

Loss of Energy. Applying the principle of work and energy to the bag and box just before and just after collision, we have

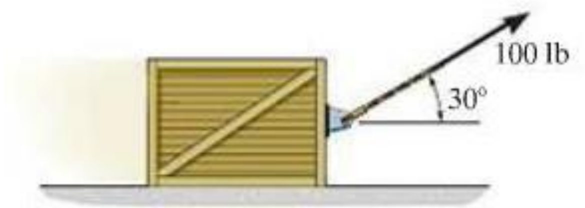
$$\Sigma U_{1-2} = T_2 - T_1;$$

$$\Sigma U_{1-2} = \left[\frac{1}{2}\left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(5.21\text{ ft/s})^2 + \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(1.74\text{ ft/s})^2\right] - \left[\frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.9\text{ ft/s})^2\right]$$

$$\Sigma U_{1-2} = -10.1\text{ ft}\cdot\text{lb} \qquad \textit{Ans.}$$

Practice Questions

1. Differentiate between central impact and oblique impact.
2. Differentiate between elastic and plastic impact.
3. Explain the term “restitution” as related impact.
4. If the coefficient of kinetic friction between the 150lb crate and the ground is $\mu_s = 0.2$, determine the speed of the crate when t= 4 s. the crate starts from rest and is towed by the 100lb force.



5. Determine the coefficient of restitution e between ball A and ball B. the velocities of A and B before and after the collision are shown